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A comparative analysis of utilizing activity-based costing and the theory of constraints for making product-mix decisions

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Abstract

Activity-based costing (ABC) and the theory of constraints (TOC) represent alternative paradigms for evaluating the economic consequences of production-related decisions. However, their application can lead to contradictory product-mix decisions. To resolve this conflict, it is frequently suggested that the TOC is appropriate for the short run, while ABC is appropriate for the longer term. This paper models the selection of a product mix with the TOC and an ABC model integrating activity-based cost with the capacity of production-related activities. The paper demonstrates that management's discretionary power over labor and overhead resources determines when the TOC and ABC lead to optimal product-mix decision. Equally important, it demonstrates that both the TOC and ABC may lead to a suboptimal product mix across a wide range of economic conditions. The paper develops a more general model of the product-mix decision and demonstrates that the TOC and ABC are special cases of this model. Finally, the paper discusses how the general model may be used to supplement information provided by the TOC and ABC.

Keywords: Activity-based cost; Theory of constraints; Product-mix decision

1. Introduction

Activity-based costing (ABC) and the theory of constraints (TOC) represent alternative paradigms to traditional cost-based accounting systems. Both paradigms are designed to overcome limitations of traditional cost-based systems and, thereby, provide more relevant information for evaluating the economic consequences of resource-allocation decisions. While their objectives are similar, the means used to achieve these objectives differ significantly. ABC models the causal relationship between products and the resources used in their production. This enables ABC to provide more accurate product-cost information for evaluating the profitability of the firm's product lines and customer base [1]. Conversely, the TOC represents an application of general systems theory for optimizing production. It uses the most constrained of the firm's activities to guide production and process improvement decisions. Firms adopting the TOC

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indicate that it has aided in reducing lead time, cycle time, and inventory, while improving productivity and quality [2].

One of the questions confronting many managers today is deciding which paradigm to select for production-related decisions. Studies comparing the decision usefulness of ABC and the TOC are contradictory. Low [3] and Spoede et al. [4], using numerical examples, illustrate that the TOC leads to a more profitable product mix than ABC. Low ([3], p. 36) noted that the "activity-based cost allocation procedure was a great deal more complex than traditional costing procedures, but it was not particularly helpful in a strategic sense". Kee [5], using a similar example, illustrates that an ABC model integrating the cost and capacity of production activities outperforms the TOC. The generalizability of these studies is limited due to their use of numerical examples to illustrate the relationship between the TOC and ABC. Consequently, more rigorous analysis is needed to assess the generalizability of the Low [3], Spoede et al. [4], and Kee [5] studies and to reconcile their contradictory conclusions.

The complementary nature of the TOC and ABC has been examined by Bakke and Hellberg [6], MacArthur [7], and Holmen [8]. They suggest that the TOC is appropriate for the short run, while ABC is appropriate for longer-term decisions. However, as noted by Bakke and Hellberg ([6], p. 13), there is no clear-cut demarcation between short-term and long-term decisions and short-term decisions may have longer-term economic consequences. Time is a surrogate in these studies for other factors in the firm's operations that determine when the TOC and ABC lead to optimal resource-allocation decisions. However, the nature and impact of these factors on ABC and the TOC were not addressed. Accordingly, determination of these factors is crucial for understanding the strengths and limitations of the TOC and ABC as information systems.

This study examines the economic conditions under which the TOC and an ABC model incorporating the cost and capacity of production activities lead to an optimal product-mix decision. It demonstrates that the TOC and ABC lead to an optimal product mix under specific sets of economic conditions. Equally important, it also illustrates that both models may lead to a suboptimal product mix across a wide range of economic activity and suggests an alternative model that may be used to supplement information provided by the TOC and ABC individually.

2. Activity-based cost and the theory of constraints

ABC is an economic model of an organization's production-related activities ([9], p. 58). The causal relationship between products and customers that consume resources is determined by tracing cost based on the factor (cost driver) that causes or correlates highly with a product's or customer's use of an activity's resources. ABC traces cost to products based on volume-related factors, such as unit-, batch-, and product-level cost drivers as well as non-volume-related cost drivers, such as product diversity, complexity, and quality. Surveys and interviews with managers using ABC indicate it is used to support a wide range of economic activities, such as product mix, pricing, and outsourcing decisions [1]. However, evidence of enhanced financial performance resulting from firms adopting ABC is somewhat limited ([1], p. 54).

Noreen [10] examined the conditions necessary for ABC to provide relevant data for dropping a product from the firm's product mix and for designing a product. For these decisions, a firm's costs must be separable into cost pools, each of which is dependent upon a single cost driver. Secondly, it requires that the cost in each pool must be proportional to the level of activity in the cost pool. Consequently, the cost function used to model each pool must be linear with a zero intercept. Finally, it requires that the activities of each cost pool must be separable with respect to the products they are used to produce. This precludes any form of dependencies between products in the production process. The conditions specified by Noreen [10] were developed for specific decision contexts. However, these conditions affect the ability of ABC to accurately trace the cost of resources to products. Consequently, the conditions necessary for ABC to provide highly accurate product cost may be quite stringent.

ABC has been criticized for its usefulness in supporting short-term decisions [11]. ABC traces the cost of resources to production activities and from activities to the products that use an activity's resources during production. Consequently, ABC is based on the resources used in production [12]. However, many of the firm's resources are contracted in advance of usage, such as rent on factory equipment, or influenced by management policy, such as retaining workers in periods of excess labor capacity. In the short run, the cost of these resources may not be controllable by the firm. Therefore, the resources used in production may not equal the resources supplied to production, i.e., the resources the firm is committed to acquiring. Consequently, in the short run, the cost behavior represented by ABC for making production-related decisions may not reflect the level of spending the firm will incur from these decisions [13]. To address this issue, Salafatinos [13] proposes using incremental analysis based on the relationship between the demand for resources and the structural level at which they are supplied. Over a sufficiently long time period, a firm may adjust its contractual relationship with suppliers and its management policies to match the use and supply of resources. Under these conditions, ABC will reflect the level of spending the firm may expect to incur from production-related decisions, i.e., the cost of resource usage is equivalent to the resources supplied to production.

ABC has also been criticized for its failure to incorporate constraints into production-related decisions [4]. Under ABC, the capacity of production activities is incorporated indirectly into the selection of a product mix. An activity's cost is divided by its practical capacity to derive a cost-driver rate for tracing cost to the products that use an activity's resources. Over a short to intermediate time horizon, a firm's management may be unable to adjust an activity's capacity to meet the firm's production needs. However, ABC fails to consider this limitation in selecting a product mix. In the long run, the capacity of production activities can be adjusted to meet the demand for the firm's products. However, even then, cost-driver rates are predicated on specific levels of production capacity. Analysis of this capacity is critical for understanding the production opportunities inherent in cost-driver rates and for evaluating whether these capacity levels are optimal for the firm. Kee [5] and Malik and Sullivan [14] demonstrate that activity-based cost may be integrated with the capacity of production activities in order to incorporate bottleneck activities into product-mix decisions. However, the basis for making these decisions remains ABC. Throughout the remainder of the paper, this approach will be used to represent the selection of a product mix with ABC.

The theory of constraints was developed by Goldratt as a process of continuous improvement [15]. The primary focus of the TOC is managing bottleneck activities that restrict the firm's performance. As noted by Goldratt ([15], p. 4), any system must have at least one constraint. The TOC consists of a set of focusing procedures for identifying a bottleneck and managing the production system with respect to this constraint, while resources are expended to relieve this limitation on the system. When a bottleneck is relieved, the firm moves to a higher level of goal attainment and one or more new bottlenecks will be encountered. The cycle of managing the firm with respect to the new bottleneck(s) is repeated, leading to successive improvements in the firm's operations and performance.

Goldratt ([16], pp. 31–40) indicates that many of the assumptions underlying traditional cost-based accounting systems, as well as ABC, are no longer valid and that these systems are leading many companies to disaster. Consequently, he proposes using an alternative measurement system to evaluate the impact of production-related decisions. The TOC is implemented through the global operational measurements of throughput, the rate at which the system generates money through sales; inventory, all money the system invests in purchasing items the system intends to sell; and operating expenses, all money the system spends turning inventory into throughput ([17], p. 29). Under this measurement system, direct material is treated as a variable cost. Conversely, labor and overhead are assumed to be resources the firm is committed to acquiring and is unable to influence ([16], pp. 36–46). Therefore, the cost of labor and overhead supplied to production is treated as a period expense. Operationally, the TOC involves maximizing throughput subject to the firm's bottleneck activities. As noted by Goldratt [16], the use of the TOC represents a paradigm shift from using cost accounting to using the TOC's global operational measures to guide production-related decisions.

Advocates of the TOC question the usefulness of cost systems such as ABC for allocating labor and overhead to products ([16], p. 40). Kaplan [9] notes that ABC is not a system for allocating cost to products more accurately. Rather, it attempts to identify factors underlying the production process that cause activities to consume resources and, thereby, incur cost. The use of volume-related cost drivers and non-volume cost drivers, such as product complexity, diversity, and quality, enable ABC to provide a powerful and rich model of the relationship between why costs are incurred in the production process and the products produced. Advocates of the TOC assert that labor and overhead are a committed cost; therefore, tracing the cost of these activities to products is irrelevant for decision making. However, labor and overhead are incurred for a reason and a well-designed activity-based cost system can be instrumental in revealing these reasons. Understanding the cost of the resources used to produce a product is crucial for understanding the economics of its production. However, indiscriminate use of ABC or any other cost system can lead to suboptimal decisions. For example, a product whose revenue is less than its activity-based cost may be beneficial for the firm to produce when the firm has excess resources that cannot be terminated or deployed elsewhere in the firms operations. Consequently, an important aspect of using ABC is understanding the economic conditions under which it leads to optimal resource allocation decisions. An extended discussion of these and related issues is provided in [3,5,6,9,12,15–19].

3. Production-mix selection

One of the applications frequently used to compare the decision usefulness of ABC and the TOC is the selection of an optimal product mix [3–5]. This decision involves determining the set of products that maximizes the firms profitability. Modeling this decision when costs are proportional to production is relatively straightforward. However, when resources are consumed by unit-, batch-, and product-level activities, the product-mix decision becomes more complicated and cumbersome to represent. To model the product mix decision, the following notation will be used:

- i product index
- j production activity index
- k production level index ($k = 1$ denotes unit-level and $k = 2$ product-level)
- X_{i1} units of product i produced
- X_{i2} determines whether product i is produced ($X_{i2} = 1$) or not ($X_{i2} = 0$)
- q_{ij1} quantity of unit-level activity j used to produce a unit of product i
- q_{ij2} quantity of product-level activity j used to produce product i
- Q_{jk} capacity of the jth activity at level k available for production
- D_i market demand for product i
- c_{i0} unit cost of raw material used to produce a unit of product i
- c_{jk} unit cost of performing the jth activity at level k
- p_i price of product i
- Z value of objective function
- X_{ik}^{ABC} optimal solution of Eq. (1)
- X_{ik}^{TOC} optimal solution of Eq. (3)
- Z_{ABC} optimal value of Z in Eq. (1)
- Z_{TOC} optimal value of Z in Eq. (3)
- R_{jk} amount of Q_{jk} subject to management control
- N_{jk} amount of Q_{jk} not subject to management control
- R_{jk}^* amount of Q_{jk} subject to management control that is used in production
- N_{jk}^* amount of Q_{jk} not subject to management control that is used in production.

The process of selecting an optimal product mix with ABC may be expressed as

$$\text{maximize } Z = \sum_i (p_i - c_{i0}) X_{i1} - \sum_{i,j,k} c_{jk} q_{ijk} X_{ik}$$

$$\text{subject to } \sum_i X_{ik} q_{ijk} \leqslant Q_{jk} \quad \forall j, k,$$

$$X_{i1} \leqslant D_i X_{i2} \quad \forall i, \tag{1}$$

$$X_{i1} \geqslant 0 \quad \forall i,$$

$$X_{i2} = 0 \text{ or } 1 \quad \forall i.$$

The first set of constraints in Eq. (1) limits the quantity of the jth activity at level k used in producing the product mix X_{ik} to the activity's available capacity. The next set of constraints reflects the demand for product i. The remaining constraints reflect the range of values that X_{ik} may assume. Let the solution to Eq. (1) be the product mix denoted by X_{ik}^{ABC}. Therefore, the profit for the optimal product mix selected with ABC would be

$$Z_{\mathrm{ABC}} = \sum_{i} (p_i - c_{i0}) X_{i1}^{\mathrm{ABC}} - \sum_{i,j,k} c_{jk} q_{ijk} X_{ik}^{\mathrm{ABC}}. \tag{2}$$

The ABC model in Eq. (1) assumes only volume-related cost drivers. Unit- and product-level activities are common to most manufacturing processes, while non-volume-related activities may vary significantly across firms, production processes, and products. Therefore, an ABC model using only volume-related factors represents a base model for comparing a product mix selected with the TOC and ABC. An ABC model based on volume-related activities is comparable to the TOC in terms of the constraints underlying the selection of an optimal product mix. Consequently, an ABC model based on volume-related cost drivers permits greater specificity in comparing a product mix selected with the TOC and ABC. However, it also represents a potential limitation of the study.

The major difference in the selection of a product mix with the TOC and ABC is how labor and overhead resources are incorporated into the decision process. As indicated in Eq. (1), ABC incorporates the cost of material, labor, and overhead resources used in producing a product to evaluate whether it should be included in an optimal product mix. This is accomplished using a cost driver to measure the quantity of labor and overhead resources of the jth activity at level k, or q_{ijk}, used in a product's production. Conversely, the TOC treats labor and overhead as a period expense. Therefore, q_{ijk} and the associated cost of labor and overhead resources used in production are excluded from evaluating whether a product should be included in an optimal product mix.

An optimal product mix may be selected with the TOC using an algorithm proposed by Fox [18] and Goldratt ([16], pp. 66–77). The algorithm identifies a firms bottleneck activity, computes throughput per unit of constrained resource, and selects products with the highest ratios until the bottleneck activity's resources are consumed. Plenert [19, p. 126] suggests Fox's procedures may be inefficient when a firm's production structure contains multiple constrained resources. He demonstrates that linear-integer programming overcomes this limitation. Furthermore, linear-integer programming generates a better solution for satisfying the TOC's goal of maximizing throughput than does the TOC [19]. The procedures of the TOC for selecting an optimal product mix are frequently defended on the basis of linear programming's inability to incorporate stochastic considerations ([3], p. 35). However, the procedures proposed by Fox [18] and Goldratt ([16], pp. 66–77) for selecting an optimal product mix with the TOC do not incorporate stochastic considerations, either. Their procedures lead to an optimal product mix when the underlying production activities are deterministic. Examples used to illustrate the selection of an optimal product mix with the TOC are based on deterministic production activities (see, for example, [3], pp. 27–28; [16], p. 67; [18], pp. 43–50). Furthermore, the TOC mitigates the stochastic effects of production activities by establishing buffer stocks of inventory in front of a production bottleneck and providing protective capacity for non-constrained stochastic activities ([16], pp. 109–115). These heuristics are used after a product mix has been selected. In the remainder of the paper, the selection of an optimal product mix with the TOC will assume deterministic production activities. This is consistent with the procedures of the TOC for selecting an optimal product mix, product-mix problems presented in the TOC literature, and with the examples used to compare the TOC and ABC [3–5]. ABC, like the TOC, selects a product mix based on the assumption that production-related activities are deterministic in nature. However, unlike the TOC, ABC does not incorporate the use of buffer stocks of inventory in front of a bottleneck and protective capacity for non-constrained activities to mitigate the effects of stochastic production activities after a product mix has been selected.

Selection of an optimal product mix with the TOC may be expressed:

$$\begin{aligned}\text{maximize}\quad & Z=\sum_i (p_i-c_{i0})X_{i1}-\sum_{j,k}c_{jk}Q_{jk}\\ \text{subject to}\quad & \sum_i X_{ik}q_{ijk}\leqslant Q_{jk}\quad \forall j,k,\\ & X_{i1}\leqslant D_iX_{i2}\quad \forall i,\\ & X_{i1}\geqslant 0\quad \forall i,\\ & X_{i2}=0 \text{ or } 1\quad \forall i.\end{aligned} \tag{3}$$

In Eq. (3), the labor and overhead resources supplied to production are treated as a period expense. Therefore, only the price of product i less the cost of direct material used in production or its throughput affects the selection of an optimal product mix. Let the product mix selected using Eq. (3) be denoted by X_{ik}^{TOC}. The profitability for the optimal product mix selected with the TOC would be

$$Z_{\text{TOC}}=\sum_i (p_i-c_{i0})X_{i1}^{\text{TOC}}-\sum_{j,k}c_{jk}Q_{jk}. \tag{4}$$

ABC and the TOC are being used to select an optimal product mix from the same underlying set of production and marketing opportunities, i.e., the production and marketing constraints from which an optimal product mix is selected are identical in Eqs. (1) and (3). However, $X_{ik}^{\text{ABC}}$and X_{ik}^{TOC} were selected subject to different assumptions about the relevance of labor and overhead cost for determining an optimal product mix. That is, $X_{ik}^{\text{ABC}}$and X_{ik}^{TOC} were selected based on different decision criteria or objective functions. Therefore, while X_{ik}^{TOC} is a solution to Eq. (1), it may not be an optimal solution. Similarly, X_{ik}^{ABC} is a feasible, but not necessarily an optimal solution to Eq. (3). Therefore, the relationship between the product mixes selected by ABC and the TOC may be stated as

$$Z_{\text{ABC}}\geqslant\sum_i (p_i-c_{i0})X_{i1}^{\text{TOC}}-\sum_{i,j,k}c_{jk}q_{ijk}X_{ik}^{\text{TOC}}, \tag{5}$$

$$Z_{\text{TOC}}\geqslant\sum_i (p_i-c_{i0})X_{i1}^{\text{ABC}}-\sum_{j,k}c_{jk}Q_{jk}. \tag{6}$$

Eq. (5) compares the product mix selected with the TOC and ABC using the objective function of Eq. (1) to compute their respective profitability. As indicated, the profitability of the product mix selected with ABC is greater than or equal to that of the TOC when both mixes are evaluated based on the resources used in production. Eq. (6) compares the product mix selected with the TOC and ABC using the objective function of Eq. (3) to compute their respective profitability. It suggests that the profitability of the product mix selected with the TOC is greater than or equal to that of ABC when both mixes are evaluated based on the labor and overhead resources supplied to production.

The relationship between the profitability of the optimal product mix selected with the TOC and that of ABC may be developed from Eqs. (5) and (6). If the cost of labor and overhead supplied to production is added to and subtracted from the right-hand side of Eq. (5), it may be restated as

$$Z_{\text{ABC}}\geqslant\left[\sum_i (p_i-c_{i0})X_{i1}^{\text{TOC}}-\sum_{j,k}c_{jk}Q_{jk}\right]+\left[\sum_{j,k}c_{jk}\left(Q_{jk}-\sum_i q_{ijk}X_{ik}^{\text{TOC}}\right)\right]. \tag{7}$$

The terms in the first set of brackets on the right-hand side of Eq. (7) represent Z_{TOC}. Z_{ABC} in Eq. (7) is based on the labor and overhead resources used in production, while Z_{TOC} is based on the labor and overhead supplied to production. The terms in the second set of brackets represent the cost of the unused resources resulting from the product mix selected with the TOC. Since the cost of the unused resources is nonnegative, the profitability of ABC is greater than or equal to the profitability of the TOC or

$$Z_{\text{ABC}}\geqslant Z_{\text{TOC}}. \tag{8}$$

Eq. (8) compares the profit of ABC and the TOC based on the assumptions that maximize the profitability of each model.

3.1. Product-mix studies

Low [3] and Spoede et al. [4] used numerical examples to compare the relationship between the TOC and ABC for selecting an optimal product mix. A product mix for the TOC was selected using the procedures outlined by Fox [18] and Goldratt ([16], pp. 66–77). The product mix for ABC was selected based on ranking each product in terms of its profitability and producing those with the highest ranking until the resources of the bottleneck activity were consumed. In effect, the opportunity cost of using a bottleneck resource was ignored in selecting a product mix with ABC. However, when an ABC model integrating the cost and capacity of production activities was used to select an optimal product mix from the Low [3] and Spoede et al. [4] data, the results remain unchanged. That is, the TOC selects a more profitable product mix than an ABC model incorporating the effect of a bottleneck activity. The results of Low [3] and Spoede et al. [4] represent a realization of the properties expressed in Eq. (6). That is, the TOC will always lead to a more or equally profitable product mix relative to ABC based on the labor and overhead resources supplied to production.

Like Low [3] and Spoede et al. [4], Kee [5] used a numerical example to illustrate the relationship between the TOC and ABC for selecting an optimal product mix. Unlike Low [3] and Spoede et al. [4], a product mix for ABC was selected using Eq. (1), while that of the TOC was selected using Eq. (3). The profitability of the product mix from ABC and the TOC was compared based on the resources used in production. The results of Kee [5] represent a realization of the properties expressed in Eq. (5), i.e., ABC will always lead to a greater or equally profitable product mix relative to the TOC based on the resources used in production. Consequently, the Low [3], Spoede et al. [4], and Kee [5] studies represent general properties of the TOC and ABC models. However, unlike the conclusions of these studies, neither the TOC nor ABC is superior to the other for all product-mix decisions. As indicated in Eqs. (5) and (6), the superiority of product-mix decisions made with the TOC and ABC are dependent upon the assumptions made about the relevance of labor and overhead for selecting an optimal product mix.

3.2. *Time horizon for using ABC and the TOC*

The assertion of Bakke and Hellberg [6], MacArthur [7], and Holmen [8] that the TOC should be used in the short run appear to be based on the assumption that, in the short run, labor and overhead resources may be difficult for management to control or influence. Alternatively, the assertion that ABC should be used in the long run appears to assume that, over a longer time horizon, management has greater flexibility with respect to managing these resources. Under these circumstances, the selection of an optimal product mix is consistent with the relationship between the TOC and ABC expressed in Eqs. (5) and (6). However, in circumstances in which management has control over labor and overhead in the short run or cannot influence these costs over an extended time period, the suggestion that the TOC should be used in the short run and ABC should be used in the long term may be misleading. Equally important, these assertions provide little practical guidance for selecting between the two paradigms in circumstances in which a firm's management has neither complete nor zero control over labor and overhead.

An examination of Eqs. (5) and (6) reveals that time is not a variable or factor determining when the TOC and ABC lead to an optimal product-mix decision. Eq. (5) indicates that ABC leads to an optimal product mix based on the resources used in production. This implies that management has the ability to redeploy unused resources to productive uses elsewhere in the firm or to terminate these resources. If unused or excess capacity cannot be redeployed or terminated, the product mix selected with ABC may be suboptimal and its profitability overstated. Conversely, Eq. (6) indicates that TOC leads to an optimal product mix based on the labor and overhead resources supplied to production. This implies that management, either through choice or contractual obligations, has little or no control over these resource levels. If unused resources could be redeployed to productive uses elsewhere within the firm or terminated, the product mix selected with the TOC may be suboptimal and its profitability understated. Consequently, management's discretionary power over labor and overhead determines when the TOC and ABC lead to an optimal product mix. Management's control over labor and overhead is generally a function of the time horizon selected. For example, the shorter the time horizon, the less control management generally has over labor and overhead resources. Conversely, the longer the time horizon selected, the more control management has, or has the ability to acquire, over labor and overhead. Consequently, managers should focus on the discretionary power they have, or can acquire, over labor and overhead resources over a given time horizon to determine when the TOC and ABC will lead to optimal product-mix decisions rather than focusing on time alone.

4. Incorporating management's degree of control over resources into product-mix decisions

The conditions for using the TOC and ABC expressed in Eqs. (5) and (6) are somewhat extreme. They imply that management has either complete control or has no control over labor and overhead resources. To understand how management's degree of control over labor and overhead affects the selection of an optimal product mix, let R_{jk} and N_{jk} represent the amount of Q_{jk} subject and not subject to management control, respectively. Furthermore, let R^*_{jk} and N^*_{jk} represent the amounts of R_{jk} and N_{jk} that are consumed in production, respectively. Capacity, or Q_{jk}, is proportional to the labor and overhead supplied to the jth activity at level k expressed in the units of its cost driver. Therefore, R_{jk} and N_{jk} reflect the portion of labor and overhead resources of the jth activity at level k that are subject and not subject to management control, respectively, while R^*_{jk} and N^*_{jk} represent the amounts of these resources that are actually used in production. These parameters, like Q_{jk}, are measured in the units of the cost driver used to trace the jth activity's resources to the products using its resources during production. Selection of an optimal product mix incorporating management's degree of control over labor and overhead may be represented:

$$\begin{aligned} &\text{maximize } Z = \sum_i (p_i - c_{i0})X_{i1} - \sum_{j,k} c_{jk}(N_{jk} + R^*_{jk}) \\ &\text{subject to } \sum_i q_{ijk}X_{ik} - N^*_{jk} - R^*_{jk} = 0 \quad \forall j,k, \\ &\qquad N^*_{jk} \leqslant N_{jk} \quad \forall j,k, \\ &\qquad R^*_{jk} \leqslant R_{jk} \quad \forall j,k, \\ &\qquad X_{i1} \leqslant D_i X_{i2} \quad \forall i, \\ &\qquad X_{i1} \geqslant 0 \quad \forall i, \\ &\qquad X_{i2} = 0 \text{ or } 1 \quad \forall i. \end{aligned} \tag{9}$$

The objective function in Eq. (9) incorporates the labor and overhead that management has no control over, N_{jk}, as a period expense and the labor and overhead over which management has control that is used in production, R^*_{jk}, as a product cost. The first set of constraints states that the labor and overhead resources used in production must equal the amount of these resources that management has no control over and control over that are used in production, i.e., N^*_{jk} and R^*_{jk}, respectively. The next two sets of constraints restrict N^*_{jk} and R^*_{jk} to the amount of labor and overhead that management has no control over and control over, respectively. The remaining constraints reflect the expected market demand for the firm's products and the range of values that X_{ik} may assume.

Eq. (9) is a more general model for selecting an optimal product mix than either the TOC or ABC. It incorporates management's degree of control over labor and overhead, N_{jk} and R_{jk}, in the selection of an optimal product mix. Throughout the remainder of the paper, Eq. (9) will be referred to as the general model to distinguish it from the TOC and ABC. The relationship between the TOC and ABC may be developed from the general model by letting the variables N_{jk} and R_{jk} reflect the assumptions of the TOC and ABC with respect to labor and overhead. A product's cost under ABC is proportional to the demand it places on the firm's resources during production. Therefore, in evaluating product-mix alternatives with ABC, an implicit assumption is being made that the cost the firm will incur from a product mix is equal to the cost of the resources used in its production. Therefore, ABC data used in product mix-decisions assumes that

$$R_{jk} = Q_{jk} \quad \text{so that} \quad N_{jk} = 0, N^*_{jk} = 0,$$

and

$$R^*_{jk} = \sum_i q_{ijk}X_{ik} \quad \forall j,k. \tag{10}$$

When these values are entered into the general model, it reduces to Eq. (1). Conversely, the TOC assumes management has no discretionary power over labor and overhead resources ([16], pp. 36–46). Therefore,

$$N_{jk} = Q_{jk} \quad \text{so that} \quad R_{jk} = 0, R^*_{jk} = 0,$$

and

$$N^{*}_{jk} = \sum_{i} q_{ijk} X_{ik} \quad \forall j, k. \tag{11}$$

When these values are entered into the general model, it simplifies to Eq. (3). In effect, ABC and TOC are special cases of the general model.

The general model can be used to identify the set of product mixes that maximize profitability as R_{jk} and N_{jk} parametrically vary from their minimum value of 0 to their maximum value of Q_{jk}, while maintaining the requirement that $R_{jk} + N_{jk} = Q_{jk}$. Profitability, or Z, in the general model may increase but cannot decrease whenever R_{jk} increases and N_{jk} correspondingly decreases. In effect, the more control a firm's management has over labor and overhead resources, the greater the potential for selecting a more profitable product mix and redeploying the excess resources that result from its production. Consequently, the highest value for the general model is realized when $R_{jk} = Q_{jk}\ \forall j, k$, which is equivalent to the product mix identified with ABC. Conversely, the general model achieves its minimum value when $R_{jk} = 0\ \forall j, k$, which is equivalent to the product mix identified with the TOC. Therefore, the profitability of the product mix selected with ABC when $R_{jk} = Q_{jk}\ \forall j, k$ is greater than or equal to that of the TOC when $R_{jk} = 0\ \forall j, k$.

When a firm's management has neither complete nor zero control over labor and overhead, i.e., when $0 < R_{jk} < Q_{jk}$ for some j, k, neither the TOC nor ABC may lead to the selection of an optimal product mix. A product mix selected with either the TOC or ABC satisfies the production and marketing constraints of the general model for any given values of R_{jk} and N_{jk}, i.e., $R_{jk} + N_{jk} = Q_{jk}$. The general model indicates that when $0 < R_{jk} < Q_{jk}$ for some j, k, the costs that are relevant for selecting an optimal product mix include the cost of direct material and the labor and overhead used in production over which management has control. However, the cost used by ABC to select an optimal product mix includes the cost of all resources used in production, while the TOC includes only the cost of direct material. ABC overstates the costs relevant for selecting an optimal product mix, while the TOC understates these costs. Therefore, both models fail to reflect the cost of production when $0 < R_{jk} < Q_{jk}$ for some j, k and may fail to select the set of products that maximize profitability.

4.1. Numerical example

To illustrate the concepts presented in the previous section, consider the product-mix example presented in Table 1. XYZ Inc. is confronted with selecting an optimal product-mix from the products, product resource requirements, and resources available for production listed in Panel I. As indicated, three activities, labor, machining, and engineering, are used in production. Products A and B require a half-hour and hour of labor, respectively. The firm has 120 assembly workers who provide 240,000 hours of production capacity in a year at an expected cost of $3,840,000. To simplify the example, overhead will be represented with machining and engineering, a unit- and product-level activity, respectively. Products A and B require an hour and half-hour of machine time, respectively. Machine capacity is 600,000 hours. Finally, to produce Products A and B, 100 and 200 engineering drawings are needed, respectively, before production of either product can begin.

Unit- and product-level cost for Products A and B is provided in Panel II. Direct material and labor costs were traced to products based on their usage of these resources. The cost driver for the machining activity is the number of machine hours used in production. The expected cost of the machining activity was divided by its capacity to get a cost per machine hour of $4.00. Machining cost was then traced to products based on the number of machine hours used in their production. As indicated in Panel II, the unit-level cost of Products A and B are $17 and $28, respectively. Unit-level cost was subtracted from the price of each product to determine its unit-level profitability. The last two rows of Panel II list the product-level cost and maximum expected annual demand for each product.

In Fig. 1, a graph of the quantity of Products A and B that can feasibly be produced is given. As indicated, the feasible area for producing Products A and B is determined by direct labor capacity. Consequently, direct labor is the most

Table 1
XYZ Inc. Operating structure and activity-based cost

Panel I: Production activities

	Product		Capacity	Cost
	A	B		
Labor hours	0.5	1	240,000	$3,840,000
Machine hours	1	0.5	600,000	$2,400,000
Engineering drawings	100	200	300	$30,000

Panel II: Activity-based cost:

	Product	
	A	B
Unit level cost		
Direct material	$5.00	$10.00
Direct labor[a]	8.00	16.00
Machine hours[b]	4.00	2.00
Total unit level cost	$17.00	$28.00
Price	$25.00	$46.00
Unit level profit	$8.00	$18.00
Product level cost[c]	$10,000	$20,000
Expected demand	500,000	300,000

[a] $3,840,000/240,000 h = $16/labor hour.
[b] $2,400,000/600,000 h = $4/machine hour.
[c] $30,000/300 drawings = $100/drawing.
Product A ($100/drawing) × (100 drawings) = $10,000.
Product B ($100/drawing) × (200 drawings) = $20,000.

constraining resource, or the firm's bottleneck activity. Furthermore, an analysis of Fig. 1 indicates that the non-bottleneck constraints do not intersect or interact with the bottleneck constraint. Therefore, a product mix can be selected with the TOC based on a product's throughput per unit of the bottleneck resource. Product-level cost prevents similar procedures being used to select a product mix for ABC. However, an analysis of the feasibility area in Fig. 1 indicates that an optimal product mix for ABC will occur at the end points of the direct labor constraint. That is, the objective function for ABC achieves its maximum value at the intersection of the axis for Product B and the direct labor constraint or the axis for Product A and the direct labor constraint. Therefore, an optimal product mix for ABC may be determined by comparing the profitability of the maximum number of Products A or B that may be produced.

In Table 2, information developed from the TOC and ABC is used to select an optimal product mix. In Panel I, each product's throughput is divided by the labor used in its production. As indicated, Product A's throughput per unit of bottleneck resource ($40) is higher than that of Product B ($36). Consequently, the firm would manufacture Product A until the resources of the bottleneck were consumed. This would lead to producing 480,000 units of Product A with an expected profit of $3,330,000 (480,000 units * $20/unit − $3,840,000 − $2,400,000 − $30,000). As indicated earlier, an optimal product mix for ABC will consist of producing either Product A or Product B. In Panel II, the maximum production of Products A and B was computed by dividing their respective labor hour

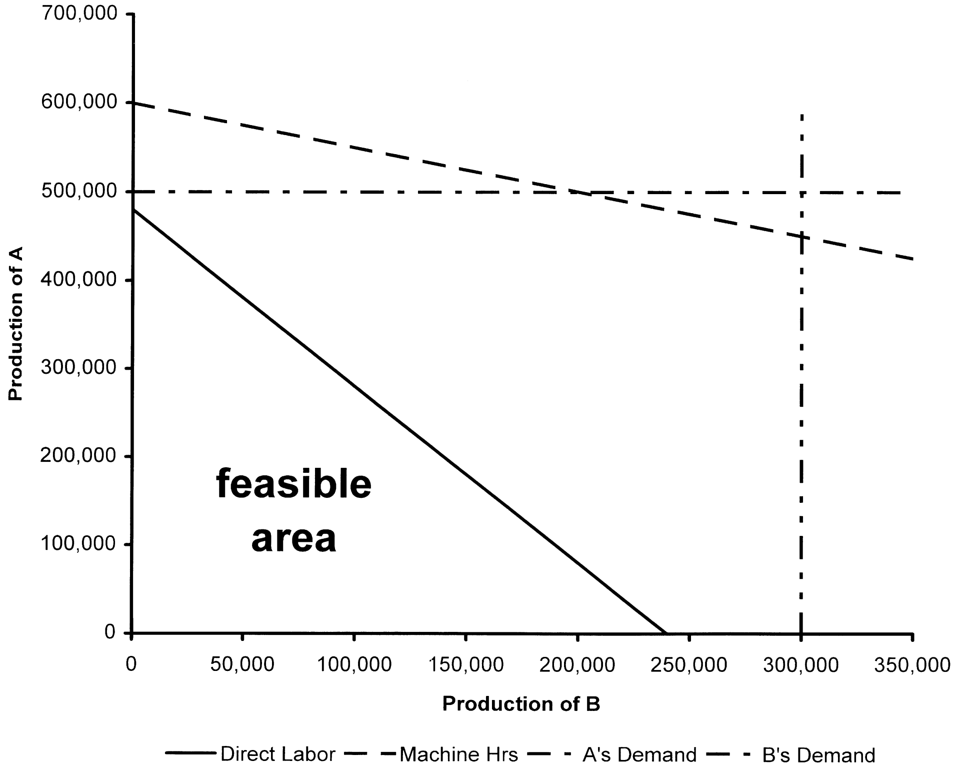

Fig. 1. Production and market constraints.

requirements into available labor capacity (see Panel I of Table 1). The total profit for each product was computed by multiplying its maximum production by its unit-level profit and subtracting its product-level cost. As indicated, Product B is expected to achieve a higher level of profitability than Product A. Therefore, under ABC, the firm would produce 240,000 units of Product B with an expected profit of $4,300,000.

In Panel III of Table 2, an income statement is presented for the product mixes selected with the TOC and ABC based on the resources used in production and the resources supplied to production. As indicated, the product mix selected with ABC leads to a higher income based on the resources used in production, while the product mix selected with the TOC leads to a higher income based on the resources supplied to production. If overhead resources are discretionary, unused machining and engineering resources can be redeployed to other uses within the firm or terminated and the product mix selected with ABC leads to an optimal product mix. Conversely, if machining and engineering resources are non-discretionary, then the firm will incur the cost of these resources regardless of the product mix selected. Under these circumstances, when the cost of unused non-discretionary resources is deducted from revenue, the product mix selected with ABC is suboptimal relative to that of the TOC. The relationship of the income of the product mixes selected using the TOC and ABC in Panel III reflects the more general properties of the TOC and ABC expressed in Eqs. (5) and (6).

As indicated previously, when the assumptions of the TOC and ABC, with respect to the discretionary nature of labor and overhead, are invalid, neither model may lead to an optimal product mix. To illustrate this assertion, assume that half of the resources of the machining function

Table 2
XYZ Inc. Product selection

	Product	
	A	B
Panel I: Product-mix selection with the TOC		
Throughput/unit	$20	$36
Labor hours/unit	0.5	1
Throughput/labor hour	$40	$36
Produce	480,000	0
Resource disposition	Used	Unused
Direct labor hours	240,000	0
Machine hours	480,000	120,000
Engineering drawings	100	200
Panel II: Product-mix selection with ABC		
Maximum production	480,000	240,000
Unit level profit	$8.00	$18.00
Total unit level profit	$3,840,000	$4,320,000
Product level cost	$10,000	$20,000
Total profit	$3,830,000	$4,300,000
Produce	0	240,000
Resources disposition	Used	Unused
Direct labor	240,000	0
Machine hours	120,000	480,000
Engineering drawings	200	100
Panel III: Product-mix income		
	TOC	ABC
Revenue	$12,000,000	$11,040,000
Cost of resources used in production		
Direct material	2,400,000	2,400,000
Direct labor	3,840,000	3,840,000
Machine hours	1,920,000	480,000
Engineering drawings	10,000	20,000
Income based on resources used in production	3,830,000	4,300,000
Cost of unused resources	500,000	1,930,000
Income based on resources supplied to production	$3,330,000	$2,370,000

are discretionary and that the remainder are non-discretionary. The general model, Eq. (9), was solved using the data in Table 1 with the discretion ary and non-discretionary machine hours equal to 300,000 to determine an optimal product mix. The product mix selected consisted of 240,000 units of Product A and 120,000 units of Product B, with an expected profit of $4,050,000. A comparison of the product mixes selected with the TOC, ABC, and the general model is provided in Table 3. In Panel I, the product mix, resources used in production, and unused resources are

Table 3
XYZ Inc. comparative analysis of product mixes selected

	TOC	ABC	General-model
Panel I: Product-mix			
Product mix			
A	480,000	0	240,000
B	0	240,000	120,000
Resources used in production			
Direct labor hours	240,000	240,000	240,000
Non-discretionary machine hours	300,000	120,000	300,000
Discretionary machine hours	180,000	0	0
Engineering drawings	100	200	300
Unused resources			
Non-discretionary machine hours	0	180,000	0
Discretionary machine hours	120,000	300,000	300,000
Engineering drawings	200	100	0
Panel II: Product-mix income			
Revenue	$12,000,000	$11,040,000	$11,520,000
Cost of resources used in production			
Direct material	2,400,000	2,400,000	2,400,000
Direct labor	3,840,000	3,840,000	3,840,000
Machine hours	1,920,000	480,000	1,200,000
Engineering drawings	10,000	20,000	30,000
Income based on resources used	3,830,000	4,300,000	4,050,000
Cost of unused non-discretionary resources	500,000[a]	720,000	0
Net income	$3,330,000	$3,580,000	$4,050,000

[a]Includes the cost of unused discretionary and non-discretionary resources consistent with the philosophy of the TOC.

compared. The unused resources are disaggregated into non-discretionary and discretionary resources. The firm can avoid the cost of unused discretionary resources, but will incur the cost of any unused non-discretionary resources. As indicated in Panel I, the unused non-discretionary machine hours for the product mixes selected with the TOC, ABC, and the general model are 0, 180,000, and 0, respectively. In Panel II, an income statement for the product mix selected with each model is given. The product mix selected with ABC leads to the highest income based on the resources used in production. However, when the cost of unused non-discretionary machine hours the firm will incur is deducted from revenue, the product mix selected with the general model leads to the highest income.

To further illustrate the relationship between the income of product mixes selected with TOC, ABC, and the general model, the discretionary number of machine hours was varied from 0 to 600,000 (0% to 100%). The income for ABC and the general model was based on the cost of resources used in production and the cost of any unused non-discretionary resources that resulted from the product mix. The income for the product mix selected with the TOC was based on the resources supplied to production consistent with its assumption about labor and overhead resources. Fig. 2 illustrates the income of the product mixes selected with the TOC and ABC as the discretionary number of machine hours varies from 0 to 600,000. The income of the product mix selected

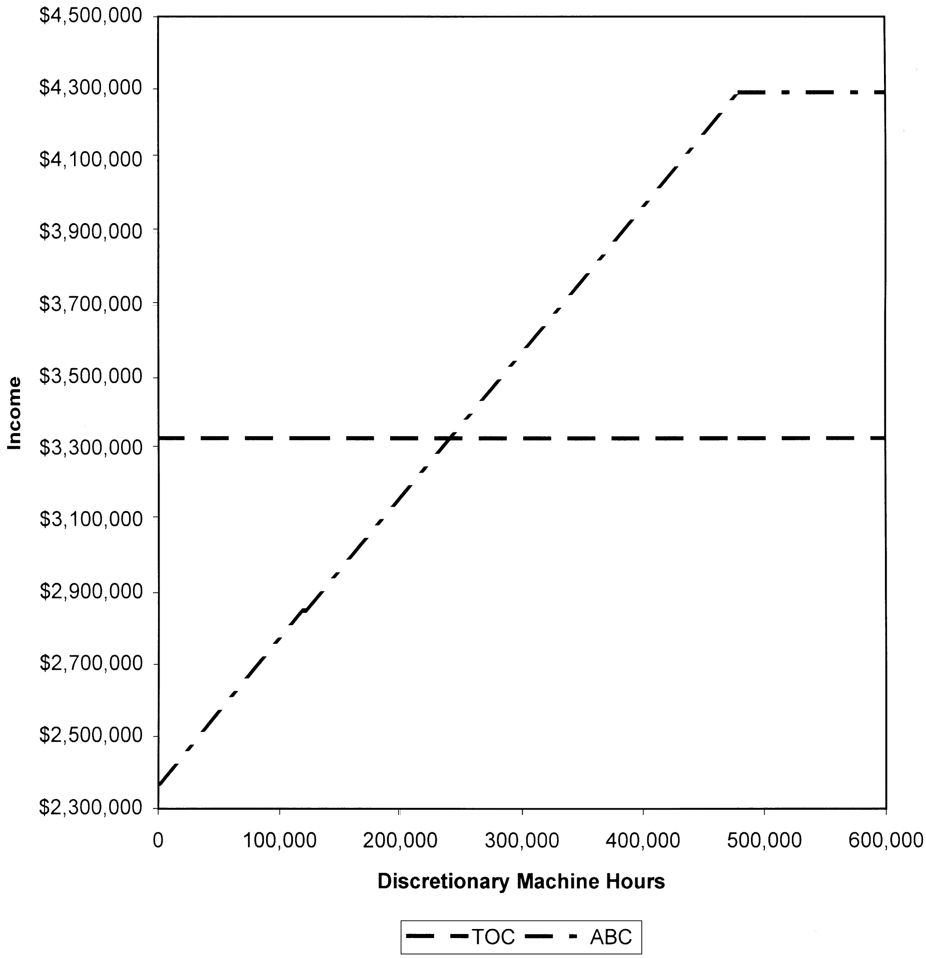

Fig. 2. Income of TOC and ABC.

with TOC is greatest relative to ABC at zero discretionary machine hours. As the number of discretionary machine hours increases, the income of the product mix selected with ABC increases monotonically. At 240,000 discretionary machine hours, the incomes of the TOC and ABC are equivalent and, thereafter, the income of ABC is greater than that of the TOC. The product mix selected with ABC uses 120,000 machine hours; therefore, the income for the product mix selected with ABC reaches its maximum value at 480,000 discretionary machine hours. An analysis of Fig. 2 indicates that the TOC and ABC may lead to selecting a product mix with a higher income relative to each other for substantial violations of their underlying assumptions with respect to the discretionary nature of labor and overhead resources.

In Fig. 3, the graph of the income for the product mixes selected with the general model as the number of discretionary machine hours varied from 0 to 600,000 is added to the graph of the income for the product mixes selected with the TOC and ABC. The income of the product mixes selected with the general model dominates that of the TOC and ABC, i.e., its income is greater than or equal to that of the TOC and ABC. The potential for the general model to select a more profitable product mix relative to the TOC and ABC results from its incorporating management's degree of control over labor and overhead resources into the decision process.

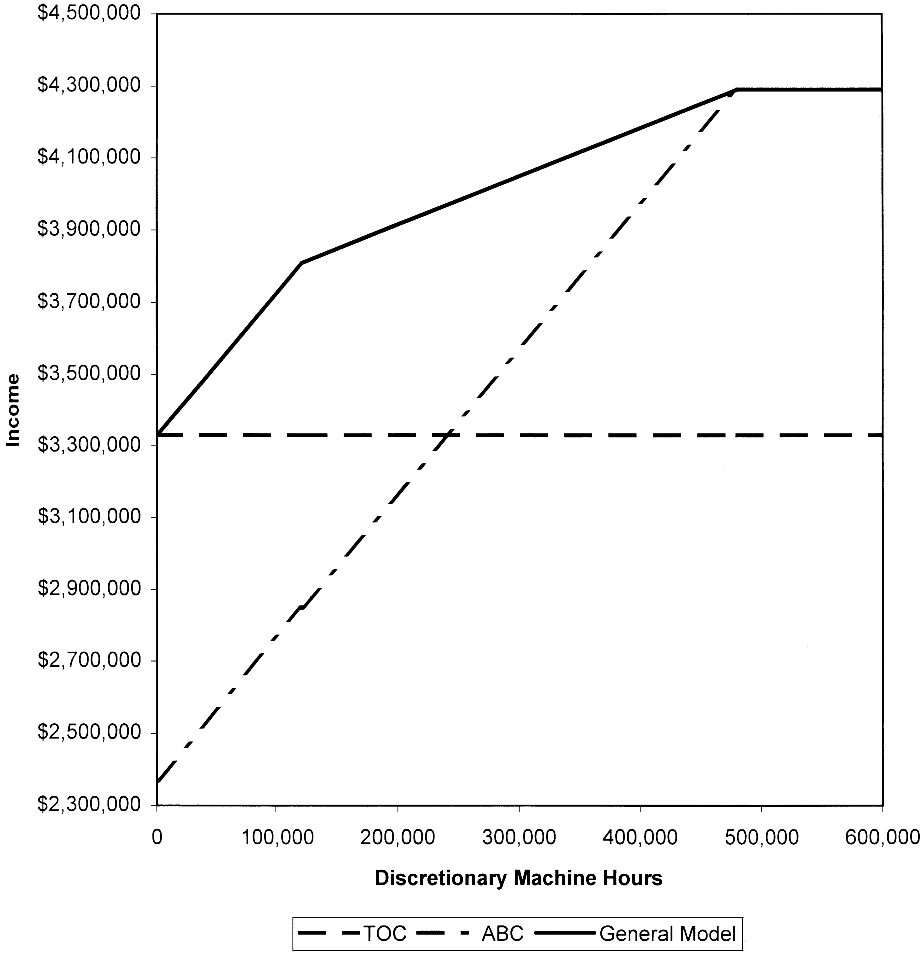

Fig. 3. Income of TOC, ABC, and general model.

As the number of discretionary machine hours in Fig. 3 approaches zero, the income of the general model approaches that of the TOC. Conversely, as the number of discretionary machine hours approaches the capacity of the machining activity, the income of the product mixes selected with the general model approaches that of ABC. This is the result of the general model reducing to the TOC and ABC when machine hours are non-discretionary and discretionary, respectively. However, as indicated in Fig. 3, when a firm's management has varying degrees of control over machining resources, the general model will lead to an optimal product mix when the TOC and ABC may not.

5. Summary and conclusion

The theory of constraints (TOC) and activity-based costing (ABC) assume that a firm's management has either no control or has complete control over its labor and overhead resources, respectively. When the respective assumptions are met, the TOC and ABC lead to optimal product mix decisions. However, when a firm has varying degrees of control over labor and overhead resources, neither the TOC nor ABC may lead to an optimal product mix. A more general model was developed in the paper that overcomes the stringent requirements of the TOC and ABC. The general model subsumes the TOC and ABC as special cases. Therefore, it may be used to supplement information from these paradigms. For example, the general model may be used to modify the TOC to reflect the economic attributes of a firm's production resources, rather than forcing these resources to fit the assumptions of the TOC. Equally important, the more general model may be used to identify a bottleneck and the unused resources in the firm's other production activities. This information may be used to identify where protective capacity may be needed to mitigate the effects of the stochastic properties of the firm's production processes. Identification of a bottleneck and the excess capacity of other production activities may be used to stimulate efforts to relieve successive bottlenecks. In effect, the general model may be used to implement the TOC's process of continuous improvement.

The product mix that maximizes the profitability of the general model occurs when a firm's management has control over labor and overhead resources, which is equivalent to the product mix identified with ABC. In effect, ABC identifies the product mix that is the most useful for the firm to produce from a strategic perspective. However, the profitability identified with ABC requires the firm to have discretionary power over its labor and overhead resources. For firms selecting a product mix with ABC based on strategic considerations, the difference in the profit of the product mix selected with ABC and the general model measures the opportunity cost of using ABC when its assumptions with respect to labor and overhead are violated. The non-discretionary resources identified from the general model represent resources the firm must gain control over to attain the profitability of the product mix identified with ABC. As the firm is able to convert non-discretionary to discretionary resources, the product mix identified with the general model approaches that of ABC.

Comparative analysis of the TOC and ABC has been restricted to the product-mix decision. However, the TOC and ABC are used across a much wider range of economic activity. Consequently, comparative analysis of the TOC and ABC for implementing a process of continuous improvement, redesigning and pricing products, outsourcing, and acquiring capital assets needs to be examined. Also, the TOC and ABC provide information for stimulating organizational learning and change; however, scant information about these implications of the TOC and ABC has been examined. Consequently, surveys and field studies of firms that have used the TOC and ABC are needed to understand how each paradigm performs in practice. An analysis of operational and financial attributes of firms that have used the TOC and ABC may be used to evaluate their relative strengths and limitations with respect to implementing a process of continuous improvement, redesigning and pricing products, outsourcing, and acquiring capital assets. Equally important, this analysis may be used to evaluate the cost and benefit of using each paradigm and its relative superiority for making different types of production-related decisions.

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